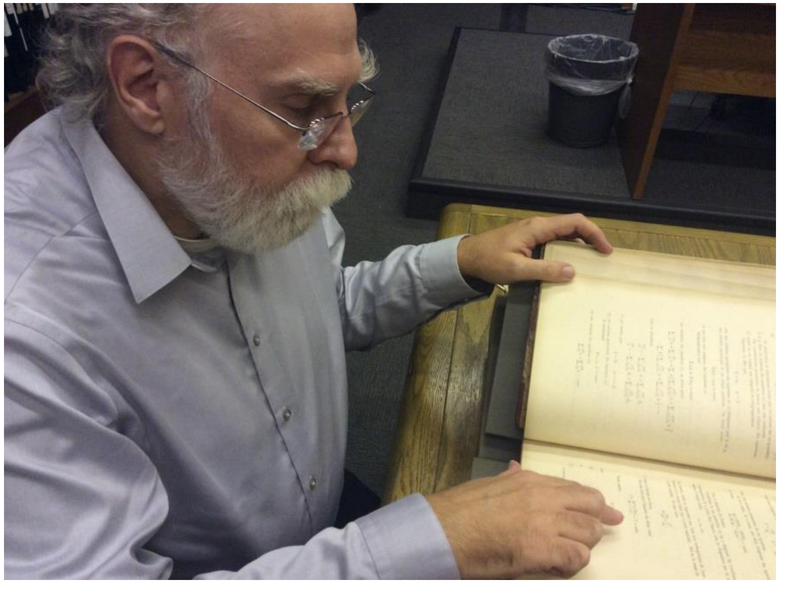
Translating Henri Poincaré

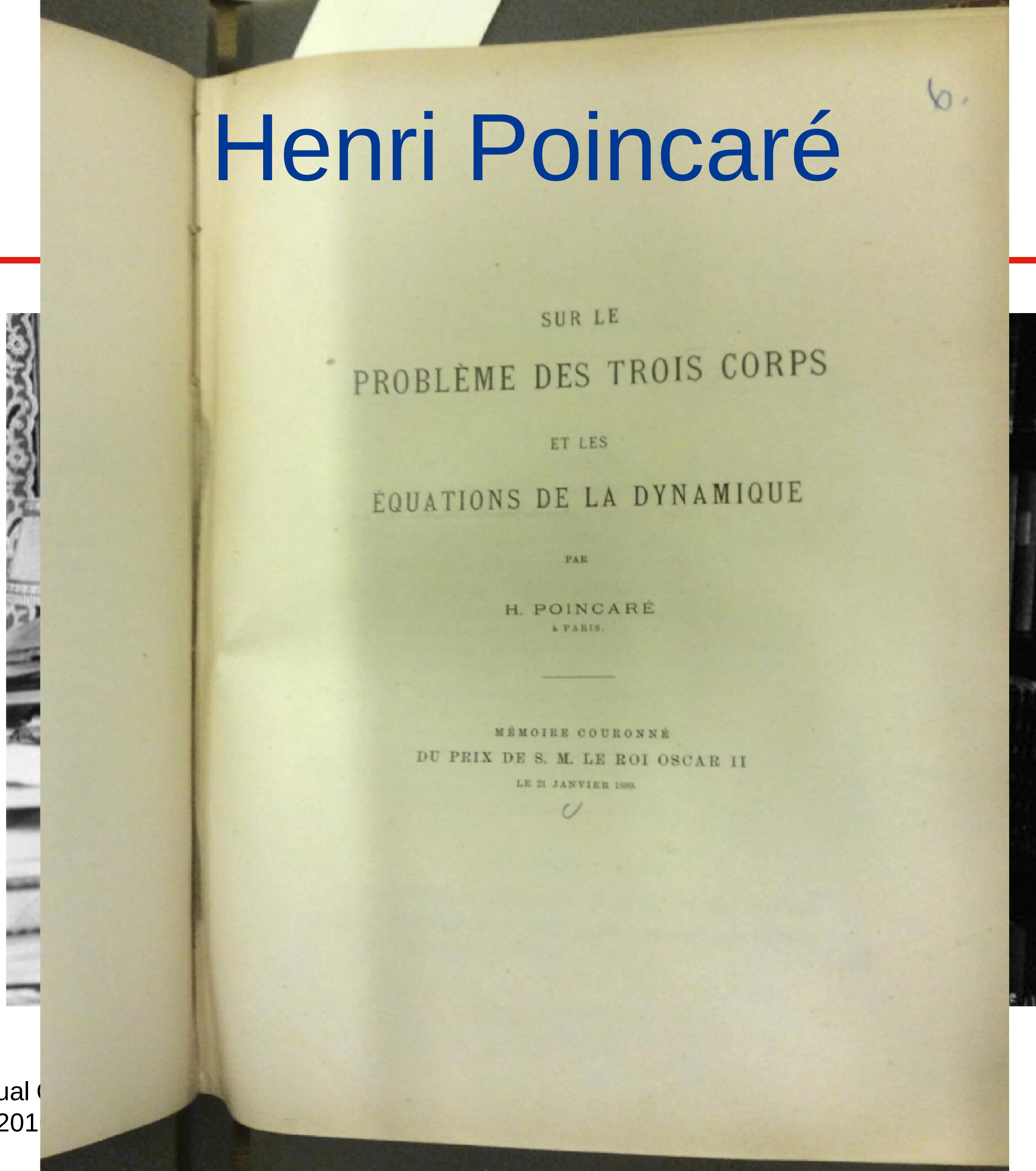
Bruce D. Popp, Ph.D.
ATA Certified Translator, Fr>En

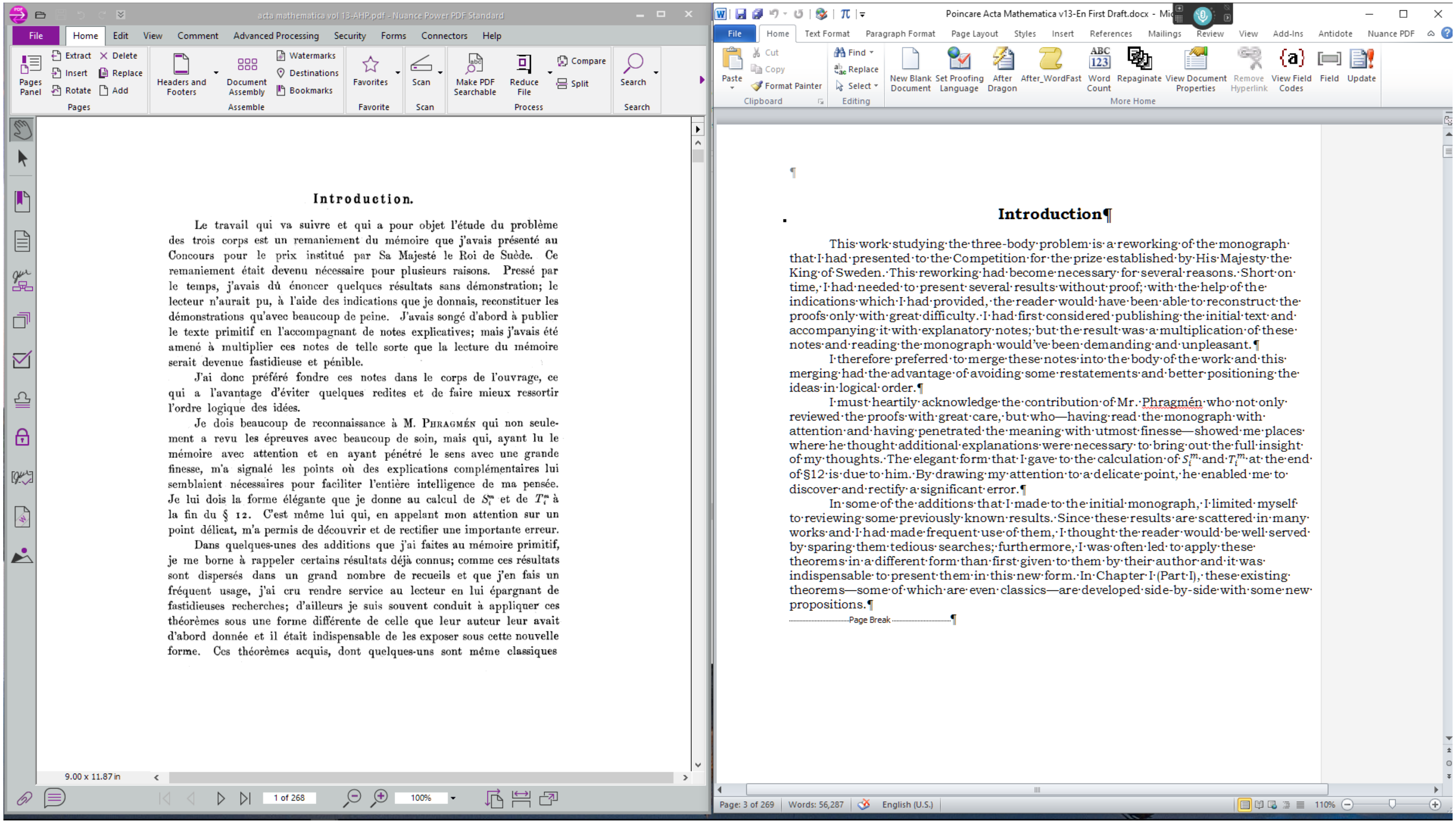
Translating Poincaré

- French, Mathematical Physics and Chaos



Henri Poincaré





Introduction.

Le travail qui va suivre et qui a pour objet l'étude du problème des trois corps est un remaniement du mémoire que j'avais présenté au Concours pour le prix institué par Sa Majesté le Roi de Suède. Ce remaniement était devenu nécessaire pour plusieurs raisons. Pressé par le temps, j'avais dû énoncer quelques résultats sans démonstration; le lecteur n'aurait pu, à l'aide des indications que je donnais, reconstituer les démonstrations qu'avec beaucoup de peine. J'avais songé d'abord à publier le texte primitif en l'accompagnant de notes explicatives; mais j'avais été amené à multiplier ces notes de telle sorte que la lecture du mémoire serait devenue fastidieuse et pénible.

J'ai donc préféré fondre ces notes dans le corps de l'ouvrage, ce qui a l'avantage d'éviter quelques redites et de faire mieux ressortir l'ordre logique des idées.

Je dois beaucoup de reconnaissance à M. Phragmén qui non seulement a revu les épreuves avec beaucoup de soin, mais qui, ayant lu le mémoire avec attention et en ayant pénétré le sens avec une grande finesse, m'a signalé les points où des explications complémentaires lui semblaient nécessaires pour faciliter l'entière intelligence de ma pensée. Je lui dois la forme élégante que je donne au calcul de S_i^m et de T_i^m à la fin du § 12. C'est même lui qui, en appelant mon attention sur un point délicat, m'a permis de découvrir et de rectifier une importante erreur.

Dans quelques-unes des additions que j'ai faites au mémoire primitif, je me borne à rappeler certains résultats déjà connus; comme ces résultats sont dispersés dans un grand nombre de recueils et que j'en fais un fréquent usage, j'ai cru rendre service au lecteur en lui épargnant de fastidieuses recherches; d'ailleurs je suis souvent conduit à appliquer ces théorèmes sous une forme différente de celle que leur auteur leur avait d'abord donnée et il était indispensable de les exposer sous cette nouvelle forme. Ces théorèmes acquis, dont quelques-uns sont même classiques

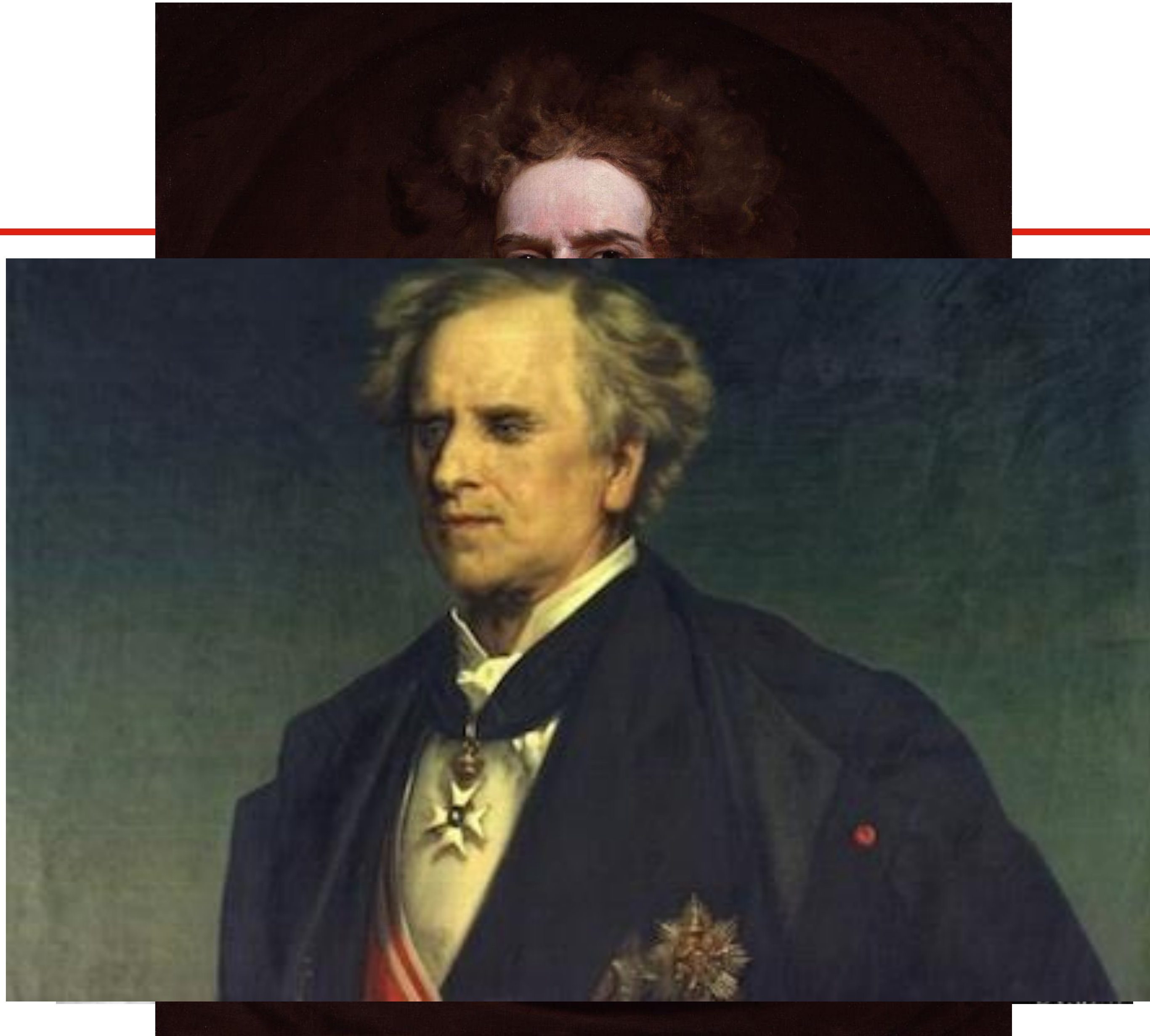
Introduction

This work studying the three-body problem is a reworking of the monograph that I had presented to the Competition for the prize established by His Majesty the King of Sweden. This reworking had become necessary for several reasons. Short on time, I had needed to present several results without proof; with the help of the indications which I had provided, the reader would have been able to reconstruct the proofs only with great difficulty. I had first considered publishing the initial text and accompanying it with explanatory notes; but the result was a multiplication of these notes and reading the monograph would've been demanding and unpleasant.

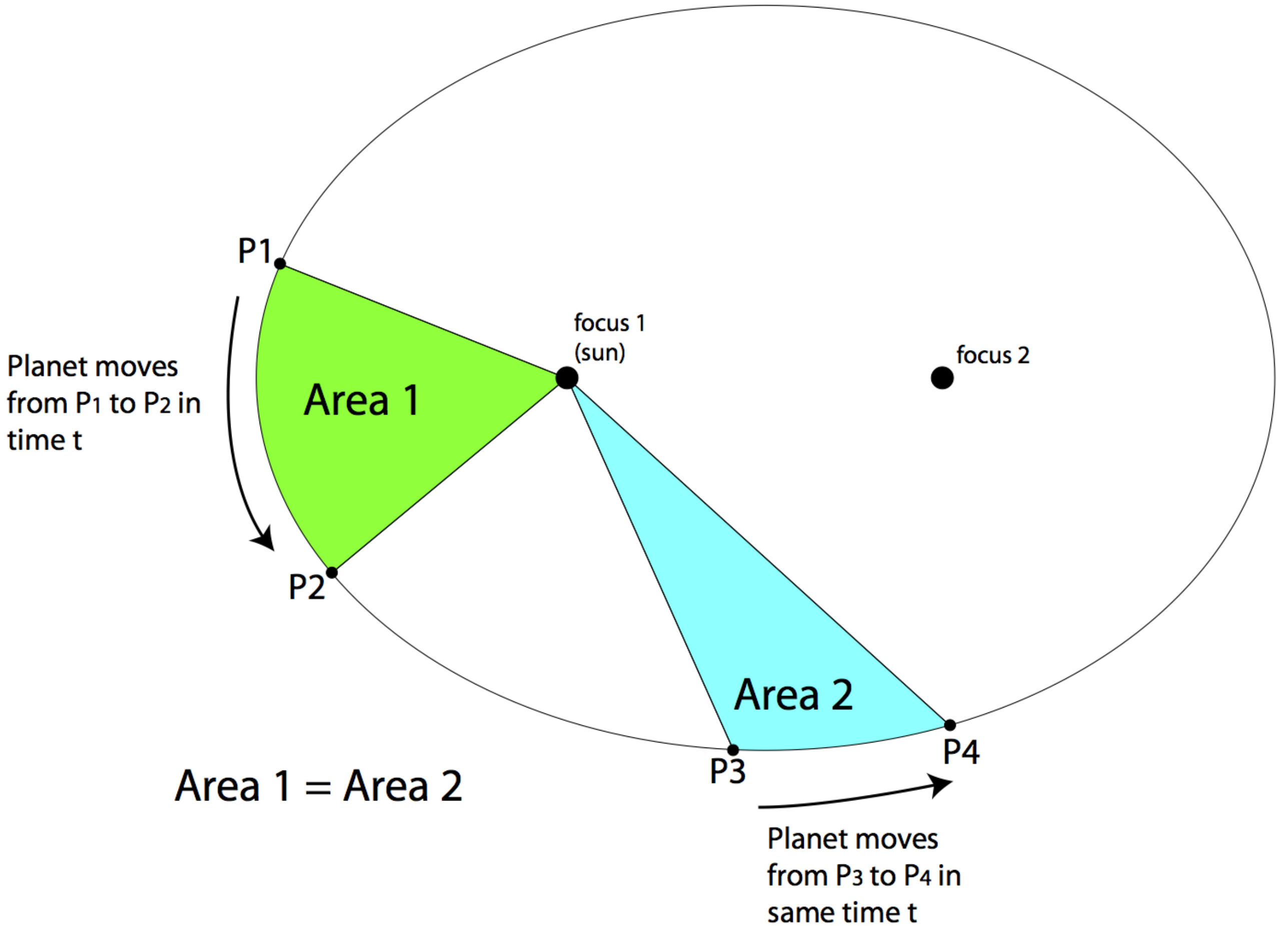
I therefore preferred to merge these notes into the body of the work and this merging had the advantage of avoiding some restatements and better positioning the ideas in logical order.

I must heartily acknowledge the contribution of Mr. Phragmén who not only reviewed the proofs with great care, but who—having read the monograph with attention and having penetrated the meaning with utmost finesse—showed me places where he thought additional explanations were necessary to bring out the full insight of my thoughts. The elegant form that I gave to the calculation of S_i^m and T_i^m at the end of §12 is due to him. By drawing my attention to a delicate point, he enabled me to discover and rectify a significant error.

In some of the additions that I made to the initial monograph, I limited myself to reviewing some previously known results. Since these results are scattered in many works and I had made frequent use of them, I thought the reader would be well served by sparing them tedious searches; furthermore, I was often led to apply these theorems in a different form than first given to them by their author and it was indispensable to present them in this new form. In Chapter I (Part I), these existing theorems—some of which are even classics—are developed side-by-side with some new propositions.



Kepler to Newton



Three Bodies

Le Verrier-Galle-Uranus

Van Flanderen and Pulkkenen

THE ASTROPHYSICAL JOURNAL SUPPLEMENT SERIES, 41:391–411, 1979 November

LOW-PRECISION FORMULAE FOR PLANETARY POSITIONS

T. C. VAN FLANDERN AND K. F. PULKKINEN
US Naval Observatory, Washington
Received 1978 November 21; accepted 1979 March 28

ABSTRACT

This paper gives low-precision (1′) formulae for geocentric and heliocentric positions of the Sun, Moon, and planets, which are valid for any epoch within 300 years of the present.

Subject headings: functions: numerical methods — planets: general

I. INTRODUCTION

Numerous modern applications have created a demand for low-precision (1′) formulae for the positions of the Sun, Moon, and planets. Examples are automatic telescope setting, spacecraft orientation, tidal theory, and planetarium projector setting. With the power of a computerized formula manipulator which can handle algebraic and trigonometric expressions, the development of simple expressions for coordinates and elements from the existing analytic theories is now feasible. This paper presents the results of such developments in a form suitable for use with hand calculators, minicomputers, or microprocessors. The series are also available on punched cards or in the form of FORTRAN subroutines. The full precision formulae (1″ or better) with unlimited time validity are being developed.

MARS (CONT.)

PLON

COEFFICIENT	T		5	7	8	13	15	16	17	19
			TRIGONOMETRIC ARGUMENTS							
-5	0	SIN	0	0	0	1	0	-3	0	0
-5	0	SIN	0	0	1	0	0	-1	0	0
-5	0	SIN	0	0	1	0	0	-2	0	0
-4	0	COS	0	0	2	0	0	-4	0	0
4	1	SIN	0	0	0	0	0	2	0	0
4	0	COS	0	0	0	0	0	0	0	1
3	0	COS	0	0	0	1	0	-3	0	0
3	0	SIN	0	0	0	0	0	2	0	-2

BETA

COEFFICIENT	T		5	7	8	13	15	16	17	19
			TRIGONOMETRIC ARGUMENTS							
6603	0	SIN	0	0	0	0	0	0	1	0
622	0	SIN	0	0	0	0	0	1	-1	0
615	0	SIN	0	0	0	0	0	1	1	0
64	0	SIN	0	0	0	0	0	2	1	0

RP

COEFFICIENT	T		5	7	8	13	15	16	17	19
			TRIGONOMETRIC ARGUMENTS							
1.53031	0	COS	0	0	0	0	0	0	0	0
-0.14170	0	COS	0	0	0	0	0	1	0	0
-0.00660	0	COS	0	0	0	0	0	2	0	0
-0.00047	0	COS	0	0	0	0	0	3	0	0

V

COEFFICIENT	T		5	7	8	13	15	16	17	19
			TRIGONOMETRIC ARGUMENTS							
0.32967	0	SIN	0	0	0	0	1	0	0	0
0.21734	0	SIN	0	1	0	0	0	0	0	0
-0.04634	0	SIN	0	0	0	0	1	-1	0	0
0.02454	0	SIN	0	0	0	0	0	0	1	0
0.01535	0	SIN	0	0	0	0	1	1	0	0
-0.00546	0	SIN	0	1	-1	0	0	0	0	0
0.00345	0	SIN	0	0	0	0	0	1	-1	0
0.00182	0	SIN	0	1	1	0	0	0	0	0
0.00114	0	SIN	0	0	0	0	0	1	1	0
0.00107	0	SIN	0	0	0	0	1	2	0	0
0.00036	0	SIN	0	0	0	0	1	-2	0	0
-0.00018	1	SIN	0	0	0	0	1	0	0	0
-0.00011	1	SIN	0	1	0	0	0	0	0	0
0.00009	0	SIN	0	0	0	0	1	3	0	0
0.00008	0	SIN	0	0	0	0	1	0	-2	0
0.00008	0	SIN	0	0	0	0	0	2	1	0
-0.00005	0	COS	0	1	0	0	0	0	0	0
-0.00004	0	COS	0	0	0	0	1	0	0	0
-0.00003	0	SIN	1	0	0	0	-1	0	0	0
-0.00003	0	SIN	0	0	0	0	0	2	-1	0
-0.00002	0	COS	0	0	0	0	1	-1	0	2
0.00002	0	SIN	0	1	2	0	0	0	0	0
0.00002	0	SIN	0	0	0	0	1	-1	0	1
-0.00002	0	SIN	1	-1	0	0	0	0	0	0
-0.00002	0	COS	0	0	0	0	1	-2	0	2
-0.00002	1	SIN	0	0	0	0	1	-1	0	0
0.00002	0	COS	0	0	0	0	1	-1	0	1
0.00002	1	SIN	0	1	-1	0	0	0	0	0

Orrery

The Prize Competition

- The problem proposed by Mittag-Leffler for a prize offered by King Oscar Sweden.

H. POINCARÉ

À PARIS.

MÉMOIRE COURONNÉ

DU PRIX DE S. M. LE ROI OSCAR II

LE 21 JANVIER 1889.

The Conclusion

Mais j'attirerai surtout l'attention du lecteur sur les résultats négatifs qui sont développés à la fin du mémoire. J'établis par exemple que le problème des trois corps ne comporte, en dehors des intégrales connues, aucune intégrale analytique et uniforme. Bien d'autres circonstances nous font prévoir que la solution complète, si jamais on peut la découvrir, exigera des instruments analytiques absolument différents de ceux que nous possédons et infiniment plus compliqués. Plus on réfléchira sur les propositions que je démontre plus loin, mieux on comprendra que ce problème présente des difficultés inouies, que l'insuccès des efforts antérieurs avait bien fait pressentir, mais dont je crois avoir mieux encore fait ressortir la nature et la grandeur.

But I would especially like to draw the reader's attention to the negative results which are developed at the end of the monograph. For example, I established that apart from known integrals the three-body problem does not comprise any analytic and one-to-one integral. Many other circumstances lead us to expect that the full solution, if it can ever be discovered, will demand analytical tools which are absolutely different from those which we have and infinitely more complicated. The more thought given to the propositions that I demonstrate later on, the better it will be understood that this problem has incredible difficulties that has certainly suggested by the lack of success in prior efforts, but I think I have brought out the nature and the immensity even better.

270 H. Poincaré. § 23.

En effet, je n'ai pu faire encore du cas particulier même auquel je me suis restreint une étude suffisamment approfondie. Ce n'est qu'après bien des recherches et des efforts que les géomètres connaîtront complètement ce domaine, où je n'ai pu faire qu'une simple reconnaissance, et qu'ils y trouveront un terrain solide d'où ils puissent s'élancer à de nouvelles conquêtes.

270 H. Poincaré §23

In fact, I have not been able to do a sufficiently in depth study with even the specific case to which I limited myself. It is only after significant research and effort that mathematicians will fully know this field, where I have only done a simple reconnaissance, and that they will find there solid ground from which they will be able to launch new conquests.

The Equations of Dynamics

- Equations of Dynamics

The equations (1) are called *canonical* when there are an even number of variables $n = 2p$, separating into two series:

$$x_1, x_2, \cdots x_p,$$
$$y_1, y_2, \cdots y_p,$$

and the equations (1) can be written:

$$\frac{dx_i}{dt} = \frac{dF}{dy_i}, \quad \frac{dy_i}{dt} = -\frac{dF}{dx_i}.$$

$$(i = 1,2, \cdots p)$$

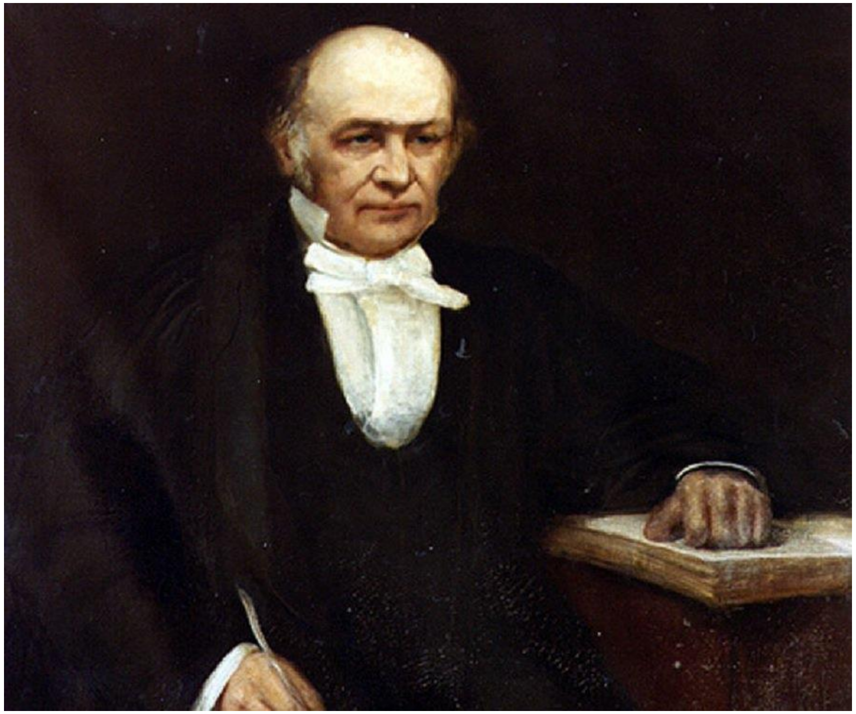
- Hamilton’s Equations

With a simple comparison, the reader can verify that these are in fact Hamilton’s equations. This involves identifying F with the Hamiltonian H, the x_i with the generalized coordinates q_i and the y_i with the generalized momentums p_i. This allows writing the canonical equations in a more familiar form:¶

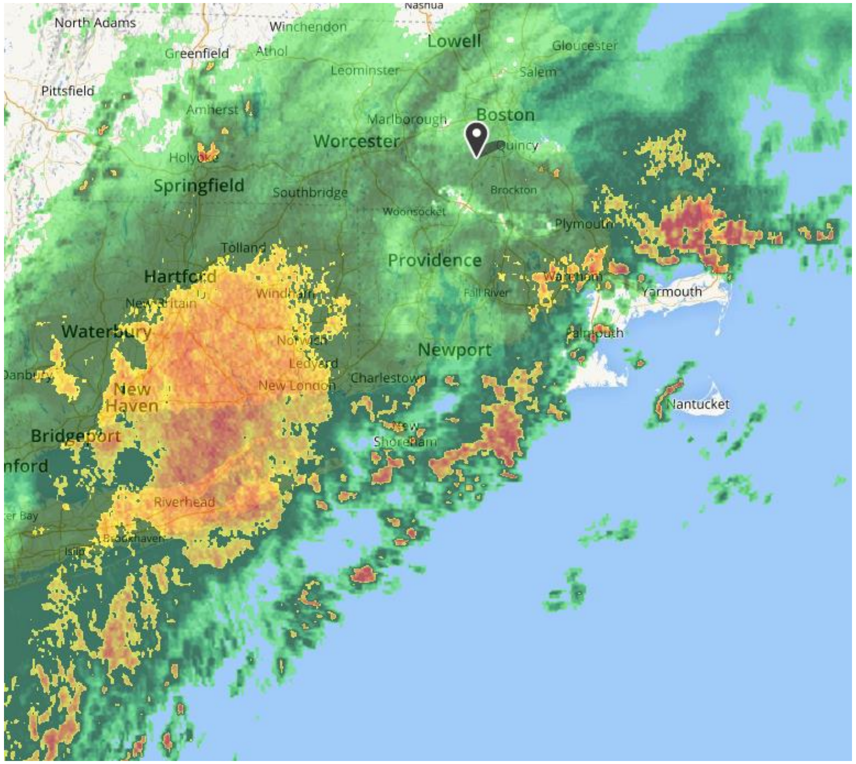
$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}.¶$$

The Hamiltonian is the total energy and as Poincaré notes at the top of page 10 it is conserved¶
An important source of generality in Poincaré’s results is his reliance on a generic Hamiltonian.

Hamiltons



Edward N. Lorenz



Deterministic, Nonperiodic

130 JOURNAL OF THE ATMOSPHERIC SCIENCES VOLUME 20

Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

Massachusetts Institute of Technology

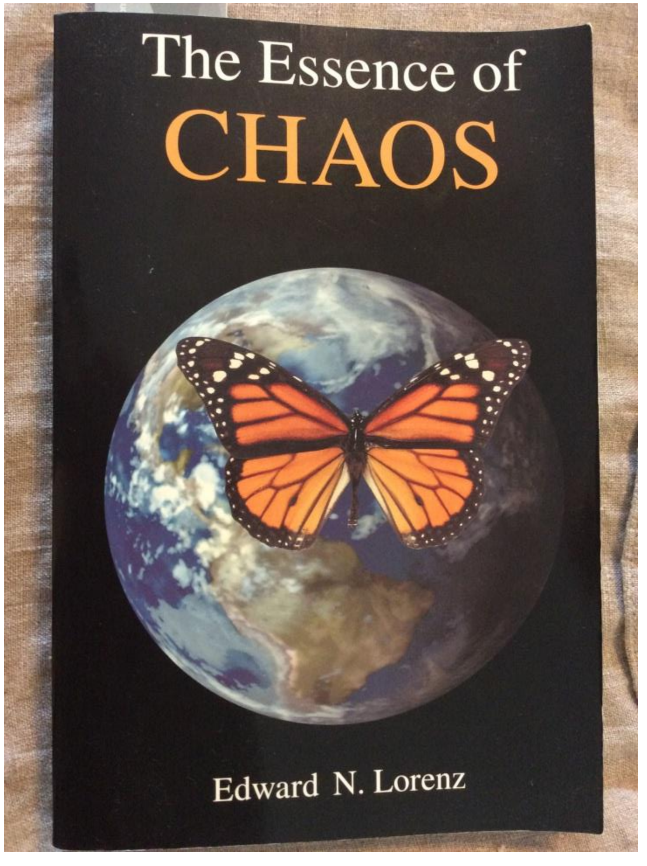
(Manuscript received 18 November 1962, in revised form 7 January 1963)

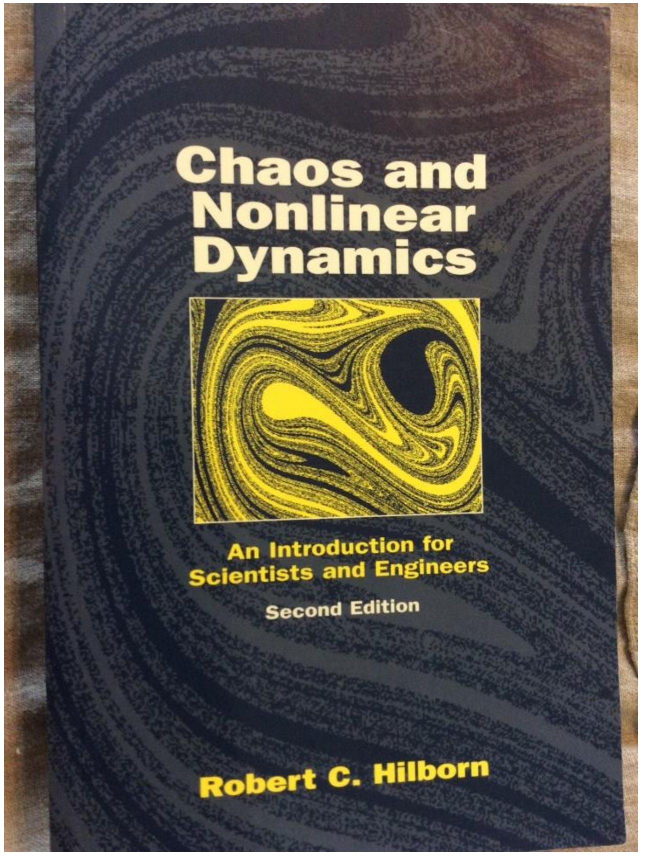
ABSTRACT

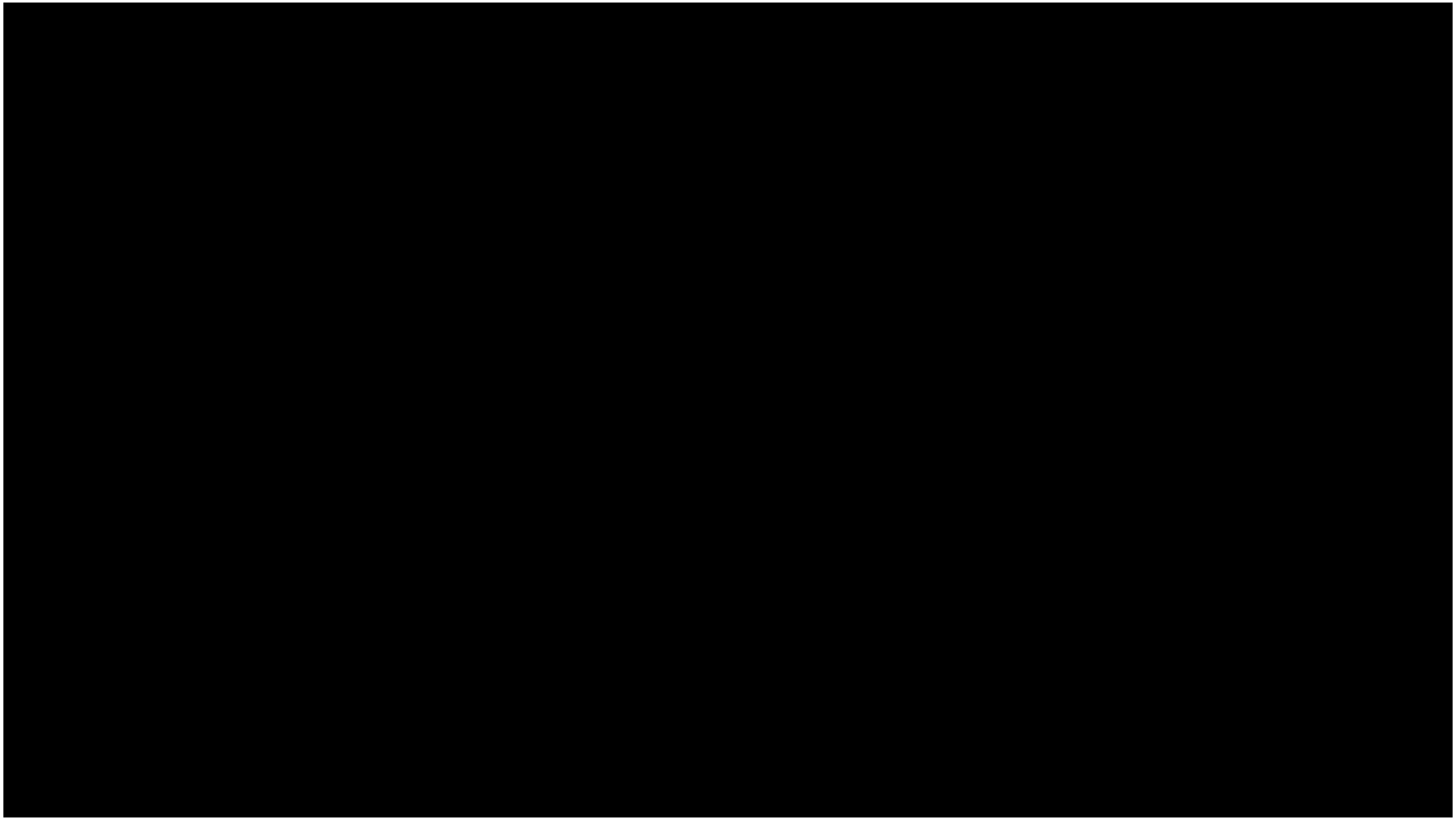
Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.


The Essence of
CHAOS
Edward N. Lorenz


Chaos and
Nonlinear
Dynamics
An Introduction for
Scientists and Engineers
Second Edition
Robert C. Hilborn



Insanity

- Doing the same thing over and over and expecting a different result isn't crazy; it's chaos.

Or is it?

88 J. Strzałko et al. / Physics Reports 469 (2008) 59–92

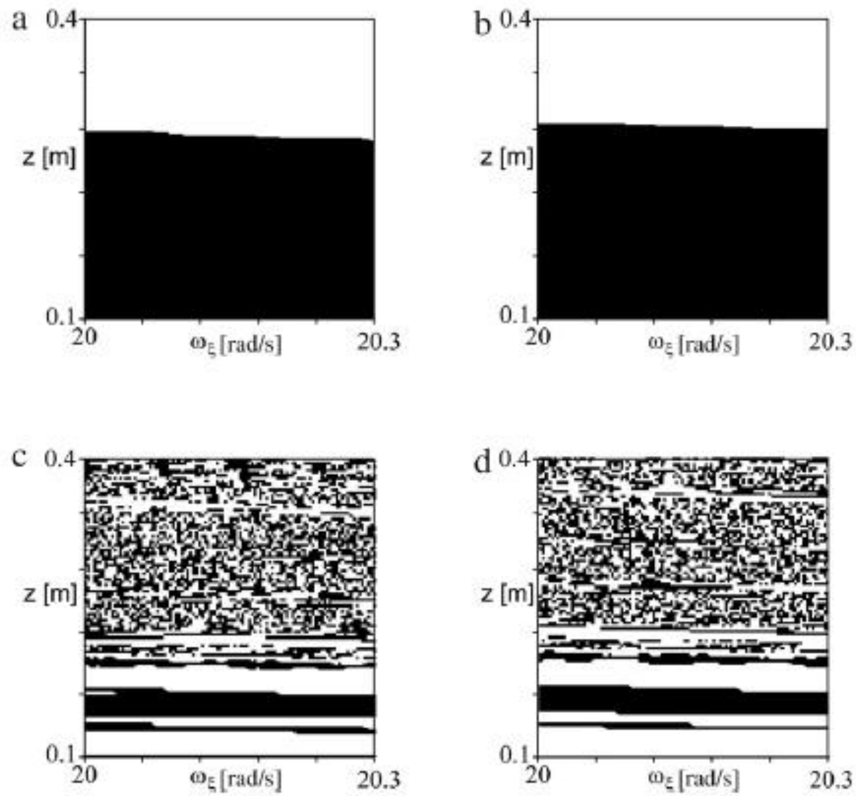


Fig. 29. Basins of attraction of heads (black) and tails (white). 2D model of an ideal thin coin described by Eqs. (37)–(39) has been simulated. (a, b) impactless motion, (c, d) motion with bouncing on the floor, (a, c) air resistance considered, $\lambda_n = 0.8$, $\lambda_\tau = 0.2$, (b, d) air resistance neglected.

Transition

- From mathematical-physics to translation

Translation Environment

et envisageons l'équation suivante:

$$(4) \qquad (\lambda_1' x_1 - Y_1')\frac{dz}{dx_1} + (\lambda_2' x_2 - Y_2')\frac{dz}{dx_2} + \dots + (\lambda_n' x_n - Y_n')\frac{dz}{dx_n} = \lambda_1' z.$$

Dans cette équation $\frac{dz}{dt}$ n'entre plus; nous pouvons donc regarder t comme un paramètre arbitraire et $x_1, x_2, \dots, x_n$ comme les seules variables indépendantes. Si donc les quantites $\lambda_1', \lambda_2', \dots, \lambda_n'$ satisfont aux conditions que nous avons énoncées plus haut, l'équation (4) (qui est de même forme que l'équation (2)) admettra une intégrale holomorphe.

Nous supposerons

$$\lambda_1' = \lambda_2' = \dots = \lambda_n'.$$

Nous supposerons de plus λ_1' réel et positif.

Cela posé, soit

$$(5) \qquad z = \sum A_{\beta.\alpha_1\alpha_2\dots\alpha_n} e^{\beta t\sqrt{-1}} x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$$

une série satisfaisant formellement à l'équation (3). Comment pourra-t-on calculer les coefficients A par récurrence.

En écrivant l'équation (3) sous la forme

$$\frac{dz}{dt} + \lambda_1 x_1 \frac{dz}{dx_1} + \dots + \lambda_n x_n \frac{dz}{dx_n} - \lambda_1 z = Y_1 \frac{dz}{dx_1} + Y_2 \frac{dz}{dx_2} + \dots + Y_n \frac{dz}{dx_n}$$

et en identifiant les deux membres on trouve:

$$A_{\beta.\alpha_1\alpha_2\dots\alpha_n}[\beta\sqrt{-1} + \lambda_1\alpha_1 + \lambda_2\alpha_2 + \dots + \lambda_n\alpha_n - \lambda_1] = P[C, A],$$

$P[C, A]$ étant un polynôme entier à coefficients positifs par rapport aux C et aux coefficients A déjà calculés.

Soit maintenant

$$(6) \qquad z = \sum A'_{\beta.\alpha_1\alpha_2\dots\alpha_n} e^{\sqrt{-1}\beta t} x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$$

une série satisfaisant à l'équation (4). Pour calculer les coefficients A' nous écrirons l'équation (4) sous la forme:

$$\lambda_1' x_1 \frac{dz}{dx_1} + \lambda_2' x_2 \frac{dz}{dx_2} + \dots + \lambda_n' x_n \frac{dz}{dx_n} - \lambda_1' z = Y_1' \frac{dz}{dx_1} + Y_2' \frac{dz}{dx_2} + \dots + Y_n' \frac{dz}{dx_n}.$$

Finally, we introduce the stream function, Ψ, for the velocities u and w in the meridional plane. Ψ is defined by $u = \partial\Psi/\partial z$ and $w = -\partial\Psi/\partial r$. Hence $\partial u/\partial z - \partial w/\partial r = \partial^2\Psi/\partial r^2 + \partial^2\Psi/\partial z^2 = \nabla^2\Psi$, and also $\partial u/\partial r + \partial w/\partial z \equiv 0$. The equations then become:

$$\frac{\delta\rho}{\rho_0} + (1+4\beta)\frac{\delta T}{T_0} + M\frac{\delta b_\varphi}{B_0} = 0 \qquad II.2a$$

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2\Psi - 2\Omega\frac{\partial v}{\partial z} + \frac{g}{\rho_0}\left(\cos\Lambda\frac{\partial}{\partial z} - \sin\Lambda\frac{\partial}{\partial r}\right)\delta\rho + \left(\frac{2V^2}{r}\right)\cdot\frac{1}{B_0}\cdot\frac{\partial\delta b_\varphi}{\partial z} = 0 \qquad II.2b$$

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)v + \frac{\Omega r}{2}|\nabla R|\left(\frac{\partial\Psi}{\partial z}\cos\Gamma - \frac{\partial\Psi}{\partial r}\sin\Gamma\right) = 0 \qquad II.2c$$

$$\frac{1}{B_0}\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)\delta b_\varphi + \frac{\Omega^2}{gM}A\left(\frac{\partial\Psi}{\partial z}\cos\Phi - \frac{\partial\Psi}{\partial r}\sin\Phi\right) = 0 \qquad II.2d$$

$$[(1+4\beta)(\gamma-1)+1]\frac{1}{T_0}\left(\frac{\partial}{\partial t} - \kappa\nabla^2\right)\delta T + \frac{(\gamma-1)M}{B_0}\cdot\frac{\partial\delta b_\varphi}{\partial t} + \frac{\Omega^2}{g}D\left(\frac{\partial\Psi}{\partial z}\cos\Delta - \frac{\partial\Psi}{\partial r}\sin\Delta\right) = 0 \qquad II.2e$$

In writing these equations nonlinear terms have been dropped. The only possible linear solutions for δb_r and δb_z are identically zero; these solutions have already been introduced into the above equations. It is straightforward to rework these equations retaining the nonaxisymmetric terms.

Previous work on magnetic buoyancy instabilities used a different set of equations than we employ here. When Schmitt and Rosner (1983) and Acheson (1978) treated the linear magnetic buoyancy problem, they used equations based on a short

16

et envisageons l'équation suivante:

$$(4)\qquad (\lambda_1' x_1 - Y_1')\frac{dz}{dx_1} + (\lambda_2' x_2 - Y_2')\frac{dz}{dx_2} + \ldots + (\lambda_n' x_n - Y_n')\frac{dz}{dx_n} = \lambda_1' z.$$

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·and·consider·the·following·equation:¶
[equation].↵
(4)¶
In·this·equation,·DZ/DT·no·longer·appears;·we·can·therefore·regard·t·as·never·arbitrary·parameter·and·x…·x·as·the·only·independent·variables.·If·therefore·the·quantities·lambda…·lambda·satisfy·the·conditions·that·we·stated·above,·the·equation·(4)·(which·has·the·same·form·is·equation·(2))·will·allow·an·analytic·integral.¶
We·will·assume:¶
[equation].¶
We·will·additionally·assume·that·lambda·is·real·and·positive.¶
Having·assumed·that,·let:¶
[equation]↵
(5)¶
be·a·series·formally·satisfying·equation·(3).·How·would·it·be·possible·to·calculate·the·coefficients·A·by·recurrence?¶
By·writing·equation·(3)·in·the·form:¶
[equation]¶
and·by·aligning·the·left-·and·right-sides,·we·find:¶
[equation],¶
where·P·[C,·A]·is·an·integer·polynomial·with·positive·coefficients·in·C·and·coefficients·A·already·calculated.¶
Now·let¶
[equation]↵
(6)¶
be·a·series·satisfying·equation·(4).·To·calculate·the·coefficients·A·prime·we·will·write·the·equation·(4)·in·the·form:¶
[equation].¶
Page Break ¶

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Page Break ¶

and consider the following equation:

[equation].
(4)

In this equation, dz/dt no longer appears; we can therefore regard t as an arbitrary parameter and $x_1, x_2, \dots x_n$ as the only independent variables. If therefore the quantities $\lambda'_1, \lambda'_2, \dots \lambda'_n$ satisfy the conditions that we stated above, the equation (4) (which has the same form as equation (2)) will allow an analytic integral.

We will assume:

[equation].

We will additionally assume that λ'_1 is real and positive.

Having assumed that, let:

[equation]
(5)

be a series formally satisfying equation (3). How would it be possible to calculate the coefficients A by recurrence?

By writing equation (3) in the form:

[equation]

and by aligning the left- and right-sides, we find:

[equation],

where $P[C, A]$ is an integer polynomial with positive coefficients in C and coefficients A already calculated.

Now let

[equation]
(6)

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[equation].

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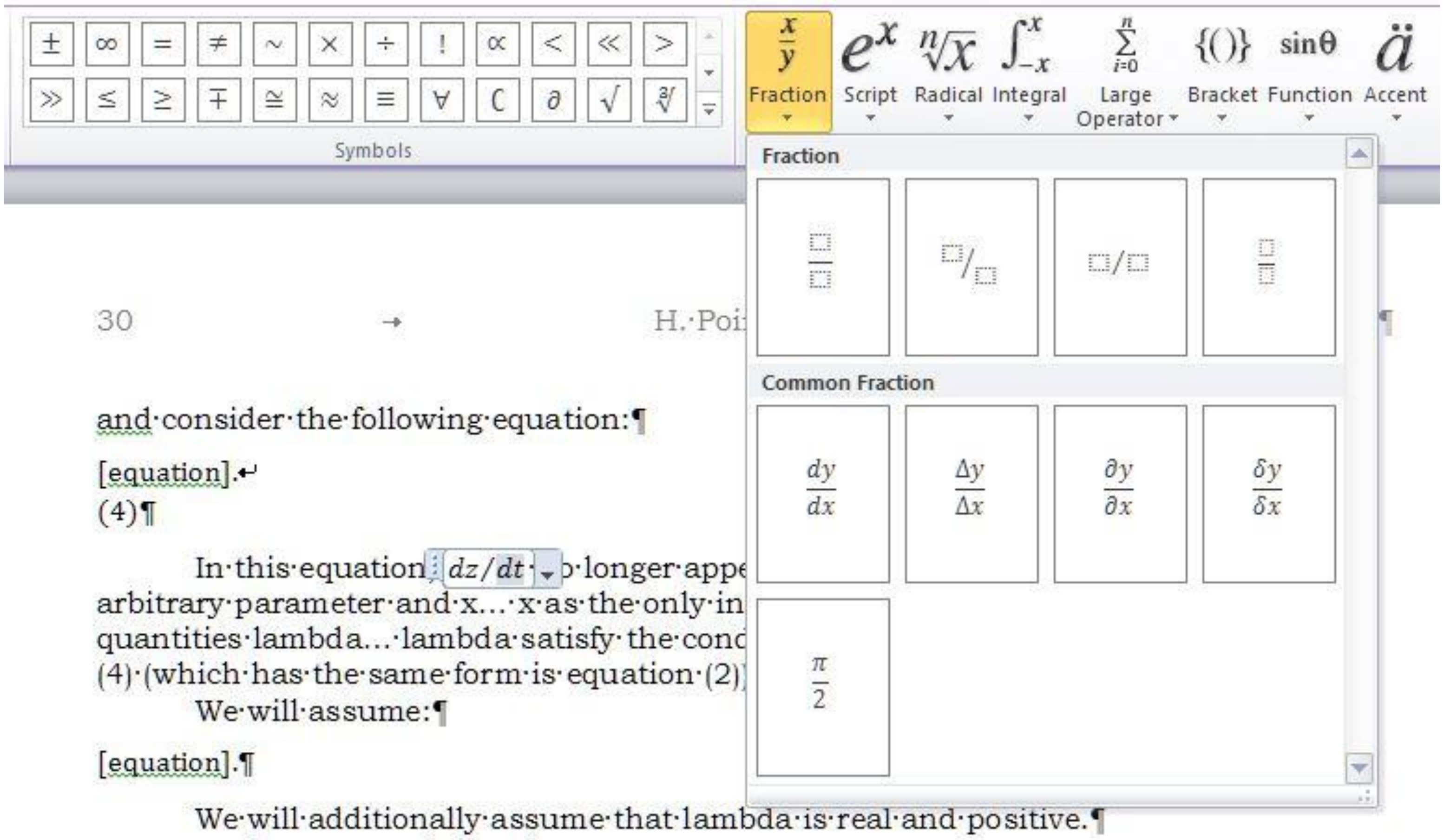
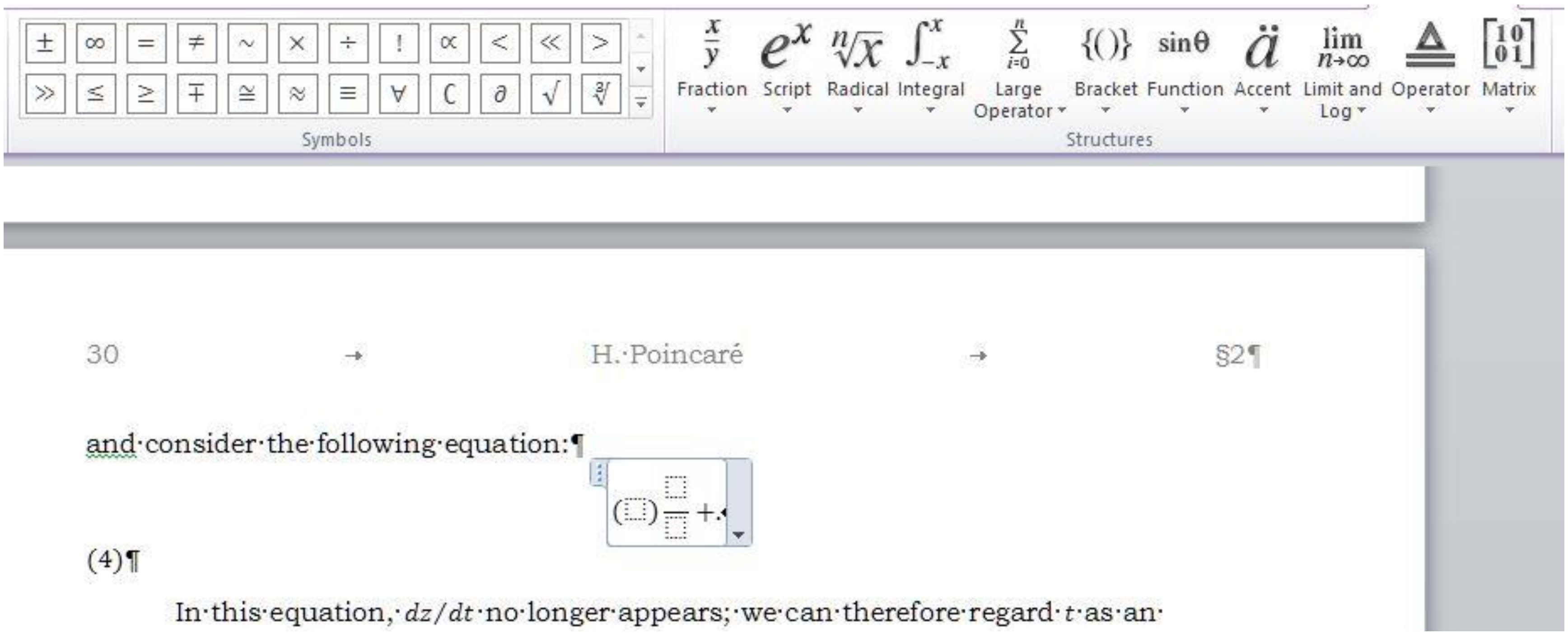
Equation Ribbon



Structures and Building Blocks



and·consider·the·following·equation:¶

$$(\square_{\square}\square_{\square} - \square_{\square})\frac{\square}{\square} + $$

(4)¶

In·this·equation,· dz/dt ·no·longer·appears;·we·can·therefore·regard· t ·as·an·

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(4)¶

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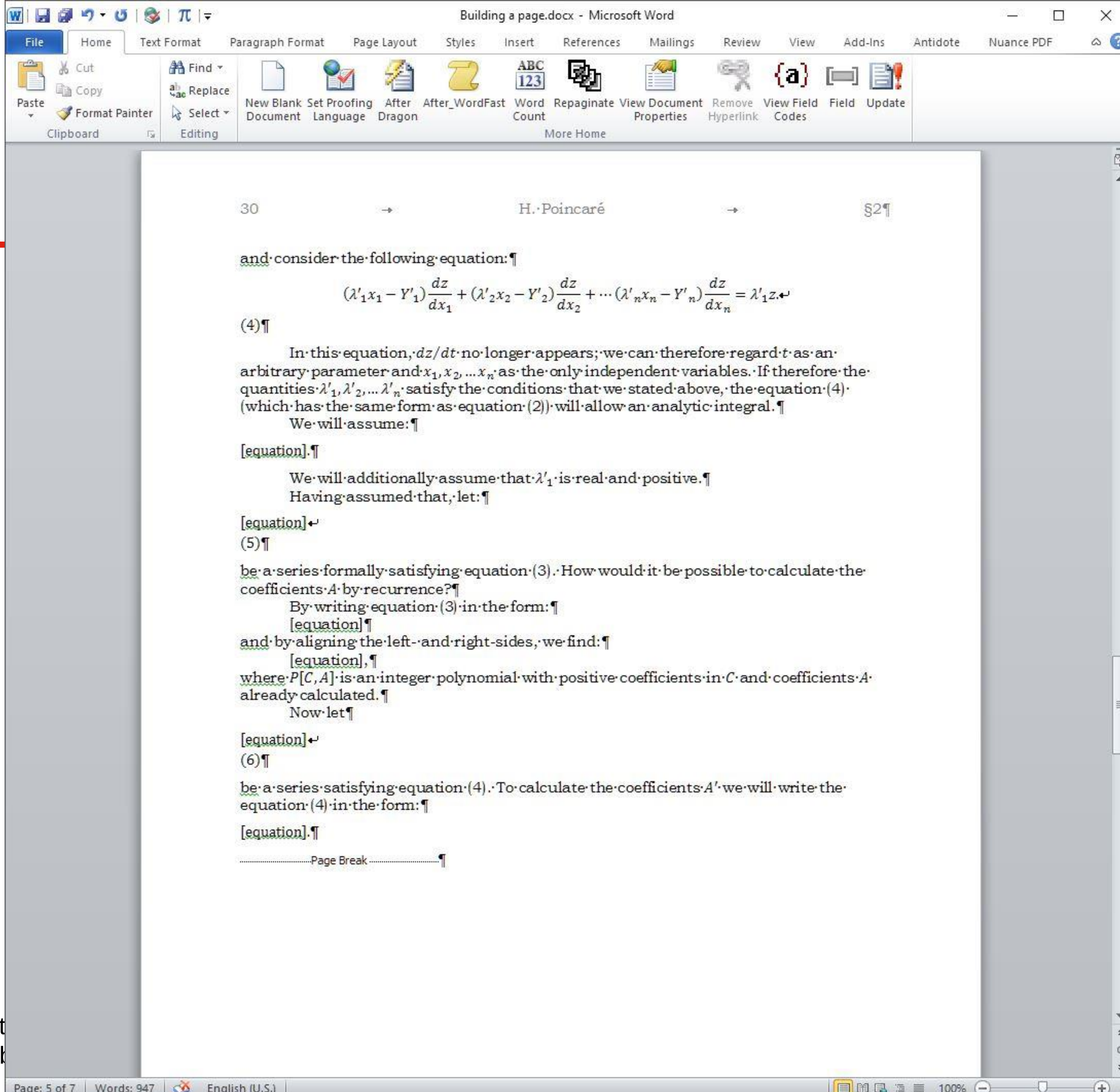

30 H. Poincaré §2

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Having assumed that, let:

$$z = \sum A_{\beta,\alpha_1,\alpha_2,\ldots,\alpha_n} e^{\beta t\sqrt{-1}} x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}$$

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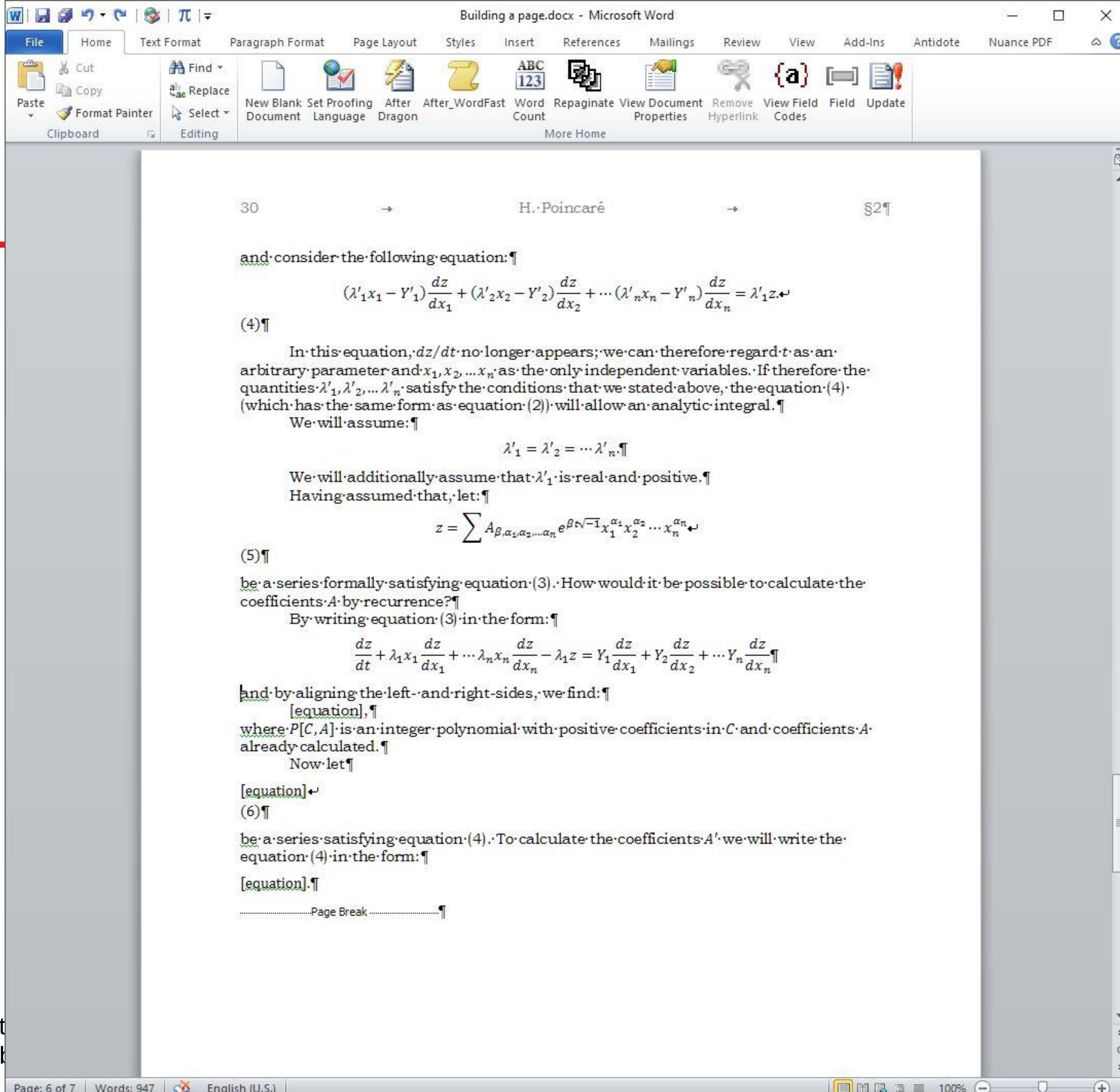

30 H. Poincaré §2

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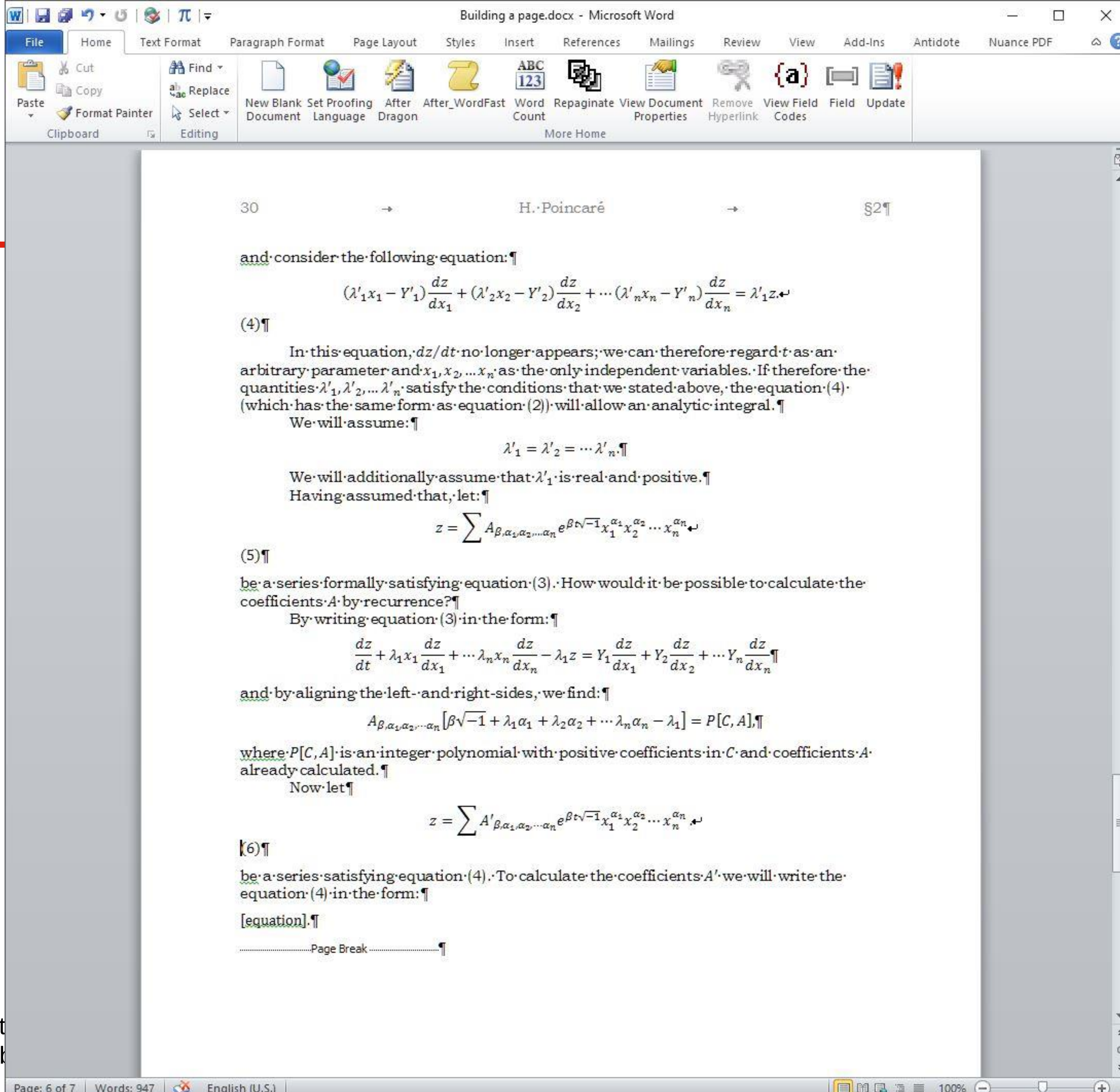

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$$A_{\beta,\alpha_1,\alpha_2,\cdots\alpha_n}\left[\beta\sqrt{-1} + \lambda_1\alpha_1 + \lambda_2\alpha_2 + \cdots \lambda_n\alpha_n - \lambda_1\right] = P[C, A],$$

where $P[C, A]$ is an integer polynomial with positive coefficients in C and coefficients A already calculated.

Now let

$$z = \sum A'_{\beta,\alpha_1,\alpha_2,\cdots\alpha_n} e^{\beta t\sqrt{-1}} x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$$

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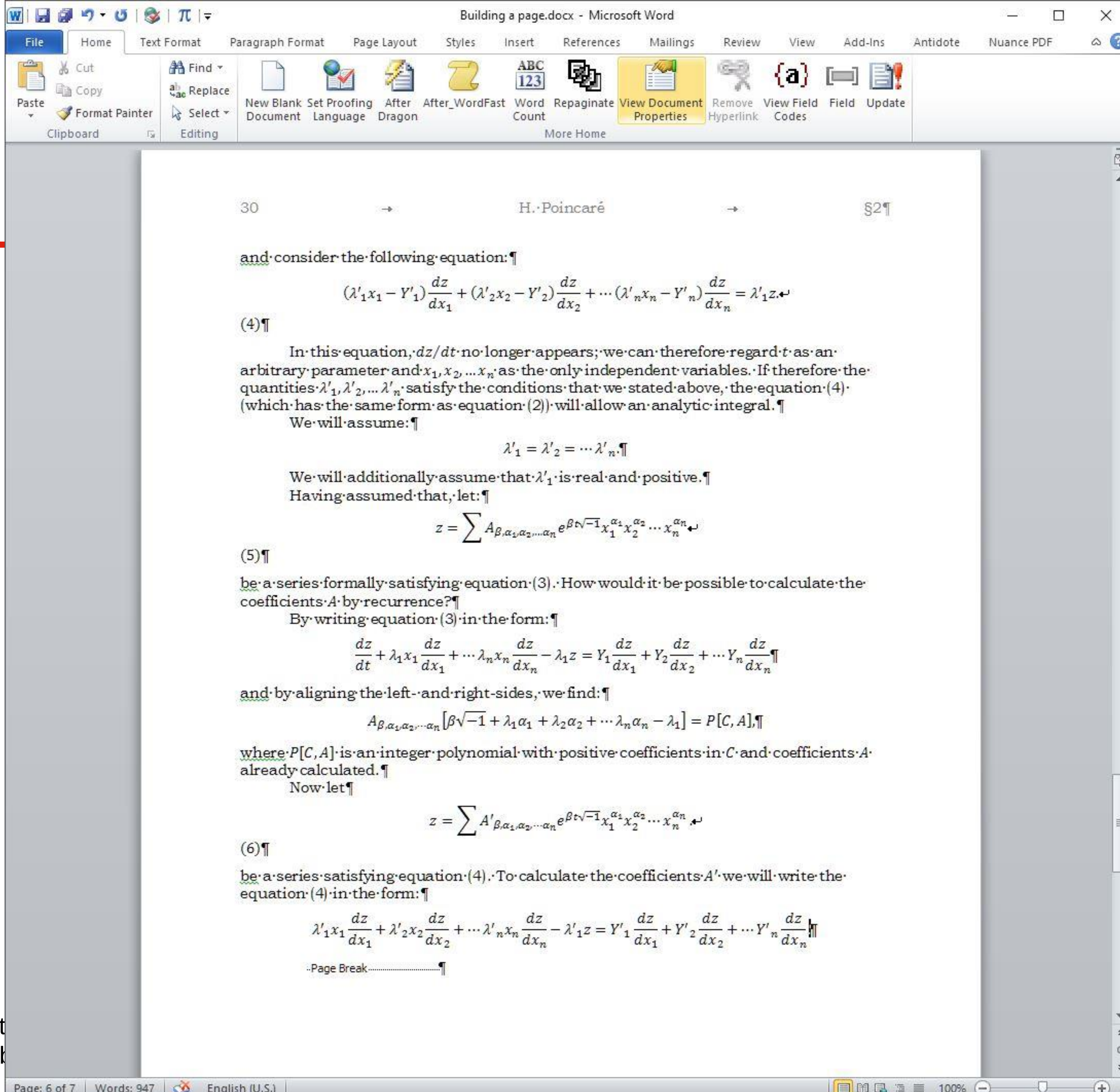

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be a series satisfying equation (4). To calculate the coefficients A' we will write the equation (4) in the form:

$$\lambda'_1 x_1 \frac{dz}{dx_1} + \lambda'_2 x_2 \frac{dz}{dx_2} + \cdots \lambda'_n x_n \frac{dz}{dx_n} - \lambda'_1 z = Y'_1 \frac{dz}{dx_1} + Y'_2 \frac{dz}{dx_2} + \cdots Y'_n \frac{dz}{dx_n}$$

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et envisageons l'équation suivante:

$$(4)\qquad (\lambda_1' x_1 - Y_1')\frac{dz}{dx_1} + (\lambda_2' x_2 - Y_2')\frac{dz}{dx_2} + \dots + (\lambda_n' x_n - Y_n')\frac{dz}{dx_n} = \lambda_1' z.$$

Dans cette équation $\frac{dz}{dt}$ n'entre plus; nous pouvons donc regarder t comme un paramètre arbitraire et $x_1, x_2, \dots, x_n$ comme les seules variables indépendantes. Si donc les quantites $\lambda_1', \lambda_2', \dots, \lambda_n'$ satisfont aux conditions que nous avons énoncées plus haut, l'équation (4) (qui est de même forme que l'équation (2)) admettra une intégrale holomorphe.

Nous supposerons

$$\lambda_1' = \lambda_2' = \dots = \lambda_n'.$$

Nous supposerons de plus λ_1' réel et positif.

Cela posé, soit

$$(5)\qquad z = \sum A_{\beta.\alpha_1\alpha_2\ldots\alpha_n} e^{\beta t\sqrt{-1}} x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$$

une série satisfaisant formellement à l'équation (3). Comment pourra-t-on calculer les coefficients A par récurrence.

En écrivant l'équation (3) sous la forme

$$\frac{dz}{dt} + \lambda_1 x_1 \frac{dz}{dx_1} + \dots + \lambda_n x_n \frac{dz}{dx_n} - \lambda_1 z = Y_1 \frac{dz}{dx_1} + Y_2 \frac{dz}{dx_2} + \dots + Y_n \frac{dz}{dx_n}$$

et en identifiant les deux membres on trouve:

$$A_{\beta.\alpha_1\alpha_2\ldots\alpha_n}[\beta\sqrt{-1} + \lambda_1\alpha_1 + \lambda_2\alpha_2 + \dots + \lambda_n\alpha_n - \lambda_1] = P[C, A],$$

$P[C, A]$ étant un polynôme entier à coefficients positifs par rapport aux C et aux coefficients A déjà calculés.

Soit maintenant

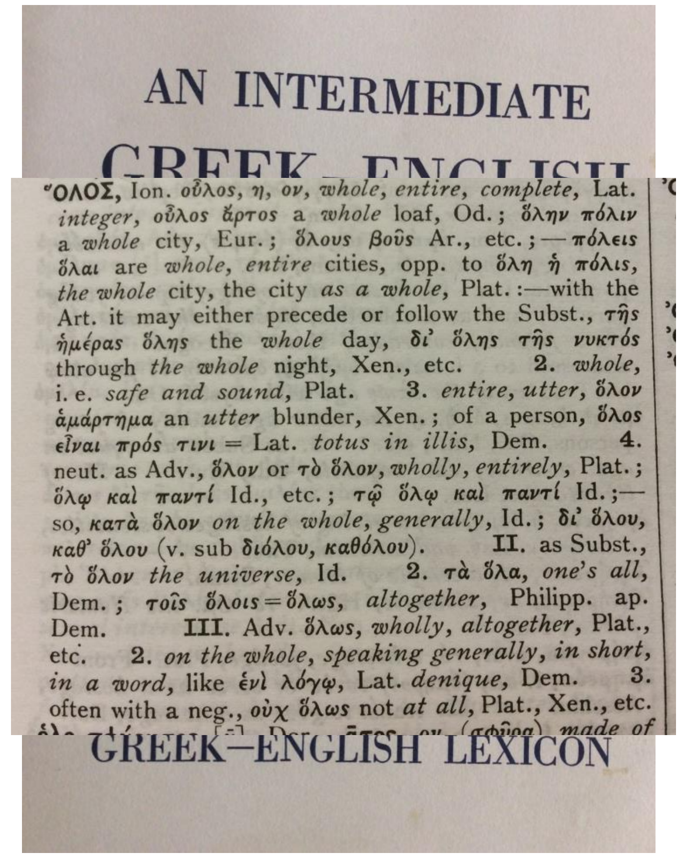
$$(6)\qquad z = \sum A'_{\beta.\alpha_1\alpha_2\ldots\alpha_n} e^{\sqrt{-1}\beta t} x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}$$

une série satisfaisant à l'équation (4). Pour calculer les coefficients A' nous écrirons l'équation (4) sous la forme:

$$\lambda_1' x_1 \frac{dz}{dx_1} + \lambda_2' x_2 \frac{dz}{dx_2} + \dots + \lambda_n' x_n \frac{dz}{dx_n} - \lambda_1' z = Y_1' \frac{dz}{dx_1} + Y_2' \frac{dz}{dx_2} + \dots + Y_n' \frac{dz}{dx_n}.$$

A Challenging Term

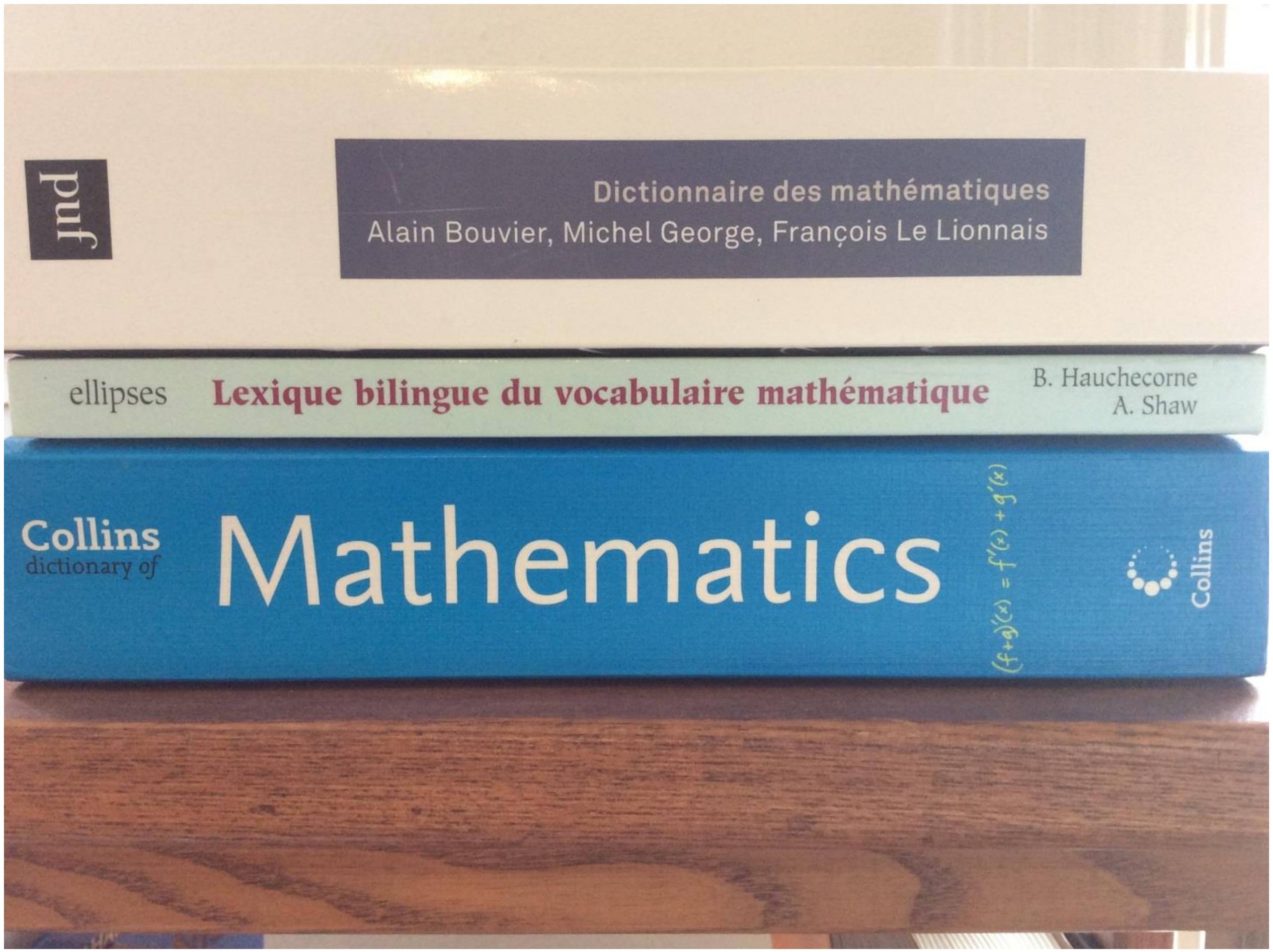
- L'équation (4)… admettra une intégrale holomorphe.



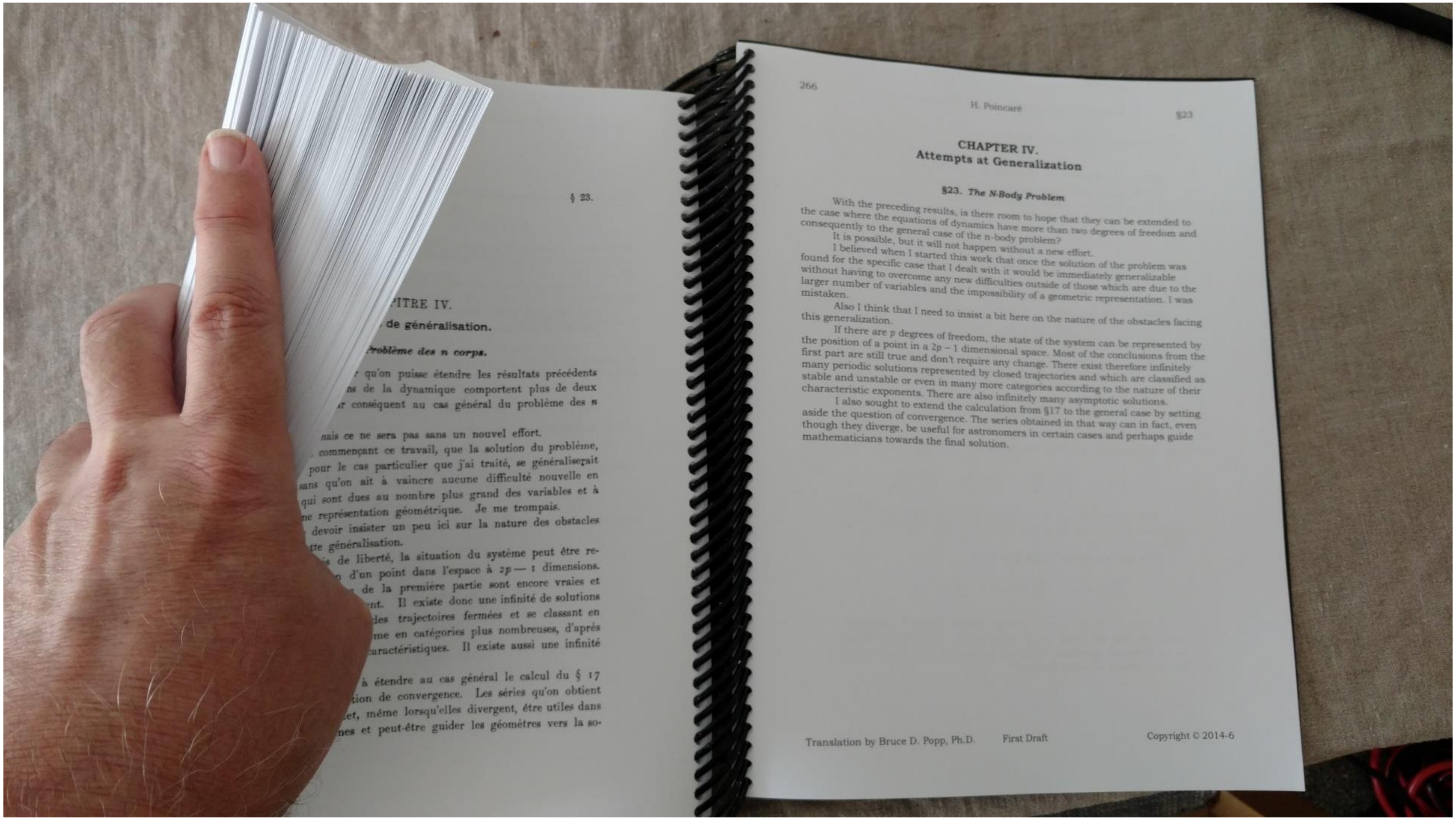
AN INTERMEDIATE GREEK–ENGLISH LEXICON

ὍΛΟΣ, Ion. *οὖλος, η, ον, whole, entire, complete*, Lat. *integer*, *οὖλος ἄρτος* a *whole* loaf, Od.; *ὅλην πόλιν* a *whole* city, Eur.; *ὅλους βοῦς* Ar., etc.;—*πόλεις ὅλαι* are *whole, entire* cities, opp. to *ὅλη ἡ πόλις*, *the whole* city, the city *as a whole*, Plat.:—with the Art. it may either precede or follow the Subst., *τῆς ἡμέρας ὅλης* the *whole* day, *δι' ὅλης τῆς νυκτός* through *the whole* night, Xen., etc. **2.** *whole*, i.e. *safe and sound*, Plat. **3.** *entire, utter*, *ὅλον ἁμάρτημα* an *utter* blunder, Xen.; of a person, *ὅλος εἶναι πρός τινι* = Lat. *totus in illis*, Dem. **4.** neut. as Adv., *ὅλον* or *τὸ ὅλον*, *wholly, entirely*, Plat.; *ὅλῳ καὶ παντί* Id., etc.; *τῷ ὅλῳ καὶ παντί* Id.;—so, *κατὰ ὅλον on the whole, generally*, Id.; *δι' ὅλου*, *καθ' ὅλου* (v. sub *διόλου, καθόλου*). **II.** as Subst., *τὸ ὅλον the universe*, Id. **2.** *τὰ ὅλα*, *one's all*, Dem.; *τοῖς ὅλοις* = *ὅλως*, *altogether*, Philipp. ap. Dem. **III.** Adv. *ὅλως*, *wholly, altogether*, Plat., etc. **2.** *on the whole, speaking generally, in short, in a word*, like *ἑνὶ λόγῳ*, Lat. *denique*, Dem. **3.** often with a neg., *οὐχ ὅλως* not *at all*, Plat., Xen., etc.

My Companions



The End


266
H. Poincaré
§23
CHAPTER IV.
Attempts at Generalization
§23. The N-Body Problem
With the preceding results, is there room to hope that they can be extended to the case where the equations of dynamics have more than two degrees of freedom and consequently to the general case of the n-body problem?
It is possible, but it will not happen without a new effort.
I believed when I started this work that once the solution of the problem was found for the specific case that I dealt with it would be immediately generalizable without having to overcome any new difficulties outside of those which are due to the larger number of variables and the impossibility of a geometric representation. I was mistaken.
Also I think that I need to insist a bit here on the nature of the obstacles facing this generalization.
If there are p degrees of freedom, the state of the system can be represented by the position of a point in a 2p − 1 dimensional space. Most of the conclusions from the first part are still true and don't require any change. There exist therefore infinitely many periodic solutions represented by closed trajectories and which are classified as stable and unstable or even in many more categories according to the nature of their characteristic exponents. There are also infinitely many asymptotic solutions.
I also sought to extend the calculation from §17 to the general case by setting aside the question of convergence. The series obtained in that way can in fact, even though they diverge, be useful for astronomers in certain cases and perhaps guide mathematicians towards the final solution.
Translation by Bruce D. Popp, Ph.D.
First Draft
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Editing

Looking for a Publisher

On the Road to Publishing